Entanglement in holographic dark energy models

R. Horvat*

Rudjer Bošković Institute, P.O.B. 180, 10002 Zagreb, Croatia

Abstract

We study a process of equilibration of holographic dark energy (HDE) with the cosmic horizon around the dark-energy dominated epoch. This process is characterized by a huge amount of information conveyed across the horizon, filling thereby a large gap in entropy between the system on the brink of experiencing a sudden collapse to a black hole and the black hole itself. At the same time, even in the absence of interaction between dark matter and dark energy, such a process marks a strong jump in the entanglement entropy, measuring the quantum-mechanical correlations between the horizon and its interior. Although the effective quantum field theory (QFT) with a peculiar relationship between the UV and IR cutoffs, a framework underlying all HDE models, may formally account for such a huge shift in the number of distinct quantum states, we show that the scope of such a framework becomes tremendously restricted, devoiding it virtually any application in other cosmological epochs or particle-physics phenomena. The problem of negative entropies for the non-phantom stuff is also discussed.

*Electronic address: horvat@lei3.irb.hr

A great variety of diverse models have been invoked to shed light on the present phase of accelerated expansion of the universe [1]. Amongst them a field-theoretical setup with the encoded information from quantum gravity, leading to a novel variable cosmological constant (CC) approach and generically dubbed that of 'holographic dark energy' (HDE), has recently triggered a lot of interest [2–4]. Besides the dark energy problem, such a framework has also proved to have a potential to shed light both on the 'old' CC problem [5] and the 'cosmic coincidence problem' [6].

A field-theoretical framework [7] underlying all HDE models describes a rather peculiar object. For a region of the size L (providing an IR cutoff) it describes a system on the brink of experiencing a sudden collapse to a black hole in that its energy density is the same as that of the black hole of the same size. As opposed to that, however, its entropy is tremendously less than the corresponding black hole entropy. If we are to describe such a system using an ordinary quantum field theory (QFT), and since black holes appear to involve a vast number of states not describable within it [8], then the QFT entropy $\sim L^3\Lambda^3$ should obey at saturation [7]

$$L^3\Lambda^3 \sim L^{3/2}M_{Pl}^{3/2} \sim (S_{BH})^{3/4} \ll S_{BH} \ , \tag{1}$$

where Λ is the UV cutoff and S_{BH} is the entropy of a black hole of the size L, $S_{BH} \sim L^2 M_{Pl}^2$. This creates thus a huge gap in entropy between the two systems having the same size and energy. Since the entropy in QFT scales extensively, it is clear that in a cosmological setting Λ should be promoted to a varying quantity (some function of L to manifest the UV/IR connection), in order (1) not to be violated during the course of the expansion. This gives the maximum energy density in the effective theory, $\rho_\Lambda \sim \Lambda^4$, to be $\rho_\Lambda \sim L^{-2}M_{Pl}^2$. Obviously, ρ_Λ is the energy density corresponding to a zero-point energy and the cutoff Λ. The main reason of why the above HDE model is so appealing in possible description of dark energy is that ρ_Λ as given above gives the right amount of dark energy in the universe at present, provided $L \simeq H^{-1}$, where H is the Hubble parameter. This also eliminates the need for fine-tuning in the 'old' CC problem [7].

Another consequence of placing the system described by (1) in a cosmological setting is the existence of event horizons induced by the energy density $\rho_\Lambda \sim L^{-2}M_{Pl}^2$. This means that our isolated system actually splits into two subsystems: the cosmological horizon and the stuff inside the horizon including the system described by (1). Obviously, the entropy

(1) of the isolated system has nothing to do with thermodynamical entropies; it rather represents the so-called fine-grained entropy, which stays exactly conserved in whatever setting the system is placed in. With the two subsystems, however, the horizon and the interior are expected to become entangled, and also thermalized sooner or later, leading to non-zero thermodynamical entropies as well as to the growth of the entanglement entropy. Thermodynamical entropies are thus additive but not conserved (owing to the generalized second law of gravitational thermodynamics), whereas the fine-grained (or entanglement) entropies are conserved but not additive (the fine-grained entropy of the whole system always stays at the value (1)).

To get a feeling of why a process of equilibration of the system (1) with the cosmological horizon is so abrupt and violent, note that an ordinary QFT is capable of describing a system at a temperature T provided

$$\Lambda \sim T >> L^{-1} \; . \tag{2}$$

On the other hand, the instantaneous horizon temperature is $T_{hor} \sim L^{-1}$, which irrespective of the choice for the IR cutoff should at present be $\sim 10^{-33}$ eV. From the relationship between the cutoffs [7]

$$\Lambda^4 \simeq L^{-2} M_{Pl}^2 \; , \tag{3}$$

one however gets $\Lambda \sim 10^{-3}$ eV. It is just this huge disparity in temperatures between the dark energy stuff and the cosmic horizon that makes this process so peculiar. In the equilibration process the UV and the IR cutoff must thus come very close together, $\Lambda \sim L^{-1}$, which does have severe consequences for the underlying QFT.

The present paper is about *internal inconsistencies* (as sketched above) inherent to *any* HDE model (independent of the choice for the IR cutoff L^{-1}) and underlied by the original theoretical framework [7]. The problem emerges when a non-black hole object like the HDE [having the same energy as the black hole for a given size but tremendously less entropy as given by (1)] is placed in a cosmological setting. Besides causing the universe to accelerate at present times, such a placing does inevitably trigger the formation of a cosmological horizon - a cosmological black hole. The root of the problem is how to thermalize an inherently non-black hole object (HDE) with the cosmic horizon (a black hole object whose entropy measures our ignorance of what lies beyond). The paper explores the various aspects of the above problem, showing, most importantly, that quantum correlations between the

horizon and the interior (consisting mostly of HDE in the dark-energy dominated epoch) and embodied in entanglement entropy turn out to be hopelessly tiny in order to trigger the thermalization process.

Below we first study the process of how the HDE gets thermalized with the cosmic horizon near the dark-energy dominated epoch purely on phenomenological grounds. Then we propose how the underlying QFT should be changed in order to account for such a violent process. Finally, we stress the restrictive scope of such a QFT. In order to avoid any influence of other components on the thermalization process, we shall expose our ideas with the aid of the non-interacting Li's model [3].

Some thermodynamical aspects of HDE models (the first and the generalized second law) have already been studied [9–14]. Usually the fluid temperature is taken to be at or proportional to the horizon temperature. Let us first set up when it is appropriate to choose so.

Here we state that thermodynamic equilibrium of the HDE with the horizon gets established if

$$\left|\frac{d_E}{\dot{d}_E}\right| \gtrsim \frac{d_E}{c_\gamma} \,, \tag{4}$$

where the future event horizon d_E is given by

$$d_E = a\int_a^\infty \frac{da}{a^2 H} \,, \tag{5}$$

with a being a scale factor. That is, departures from de Sitter space should be small enough so that the RHE of (4) is always larger than the light-crossing time of the radius d_E. Thermodynamic equilibrium having once been established at such time, it continues to exist provided the heat capacity for the whole system is positive-definite [9, 15]. Since the heat capacity of the horizon is negative, the heat capacity of the dark energy fluid should be positive (and larger in absolute value).

Let us now see how the above postulates work for the popular Li's model [3]. In a two-component universe ρ_Λ evolution is governed by [3, 15]

$$\Omega_\Lambda' = \Omega_\Lambda^2(1-\Omega_\Lambda)\left[\frac{1}{\Omega_\Lambda} + \frac{2}{c\sqrt{\Omega_\Lambda}}\right] \,, \tag{6}$$

where the prime denotes the derivative with respect to lna. In (6) $\Omega_\Lambda = \rho_\Lambda/\rho_{crit}$, where ρ_{crit} is the critical density and ρ_Λ was parametrized as $\rho_\Lambda = (3/8\pi)c^2 M_{Pl}^2 L^{-2}$, with a constant

parameter c of order one and $L = d_E$. Also $\Omega_\Lambda + \Omega_X = 1$, with X being either matter or radiation. Combining (6) with (4) for the matter case one arrives at

$$\left| \frac{\sqrt{\Omega_\Lambda}}{c - \sqrt{\Omega_\Lambda}} \right| \gtrsim 1 \ . \tag{7}$$

Employing $c = 1$[1], one obtains $\Omega_\Lambda > 1/4$. This is what is to be expected: the HDE enters the thermodynamic equilibrium with the horizon somewhere around the onset of the dark-energy dominated epoch. To see that this is by no means so for earlier cosmological epochs, we note that (6) is also capable of describing epochs where ρ_Λ occupies only a tiny fraction of the total energy density. In particular, in that limit $\Omega_\Lambda << 1$, the solution of (6) in the radiation-dominated universe reads

$$L(a) \simeq M_{Pl} \, \rho_{rad0}^{-1/2} \, a^{3/2} \ , \tag{8}$$

where ρ_{rad0} denotes the radiation energy density at the present time. Using (8) one can be easily convinced that (4) is far from being satisfied wherever in the radiation-dominated epoch of the universe.

Hence if we trust the qualitative criterion (4), then the HDE becomes thermalized with the horizon near the onset of the dark-energy dominated epoch. This means equalizing of the temperatures, that is, rapprochement of the cutoffs, Λ and L^{-1}. But it is obvious right away that the theoretical setup as given by (1) and (3) is not capable to support this scenario. By setting $\Lambda \sim L^{-1}$ in (3), one gets $\Lambda \sim L^{-1} \sim M_{Pl}$, and we need this not in the Planck-time era but some 10^{60} Planck times later.

There is also a more physical argument against (1) and (3): the entanglement entropy. The entanglement entropy in the present context would measure quantum-mechanical correlations between the horizon and its interior. As soon as the physical horizon forms, and consequently an interior observer lacks any information about the space outside the horizon, both the horizon entropy (the black hole entropy) and the entanglement entropy become nonzero. When the overall state is pure or near-pure, the entanglement entropy should behave nonextensively, that is, should depend only on the surface of the horizon separating the

[1] Although a restriction on c^2 under the combined phenomenological constraints obtained recently [17] slightly favor phantom behavior ($c^2 < 1$), our entropic arguments favor $c = 1$ (see below). Anyhow $c = 1$ taken in (7) is for the illustration purposes only.

interior from the rest. On the other hand, quantum correlations between the subsystems in any local QFT are taken care of by the UV cutoff. Consequently, we have

$$S_{ent} \sim \Lambda^2 L^2 \, . \tag{9}$$

Using (3) one arrives at

$$S_{ent} \sim \; L M_{Pl} \, . \tag{10}$$

Comparing (10) with other two type of entropies involved in the problem, $S_{HDE} \sim L^{3/2} M_{Pl}^{3/2}$ and that of black holes $S_{BH} \sim L^2 M_{Pl}^2$, S_{ent} is by far the least one near the present epoch, leading to the prominent hierarchy

$$L M_{Pl} << L^{3/2} M_{Pl}^{3/2} << L^2 M_{Pl}^2 \, . \tag{11}$$

The physical interpretation of (11) is pretty obvious. The two subsystems do interact extremely weakly so that the thermalization process cannot be initiated. Obviously, the thermodynamics of the HDE models near the present epoch is not possible within the original theoretical framework [7].

Now we can ask: is it possible to change the original QFT framework to account for a huge transfer of entropy across the horizon, needed to start a thermalization process? One possibility [18], staying purely within the realm of effective QFT, is to develop the original theory with a large number of particle species, $N >> 1$. The basic idea is that with $N >> 1$ the energy density $\rho_\Lambda \sim L^{-2} M_{Pl}^2$ stays intact, whereas both entropies, S_{ent} and S_{HDE}, now become some increasing functions of the number of field species. When the maximal allowable limit for N is approached, both S_{ent} and S_{HDE} begin to sustain the black hole entropy, $L^2 M_{Pl}^2$. The huge gap in entropies between the HDE object and black holes is thus being populated when N is increased. With $N >> 1$ (3) now gets modified to

$$N \Lambda^4 \simeq L^{-2} M_{Pl}^2 \, . \tag{12}$$

Using a criterion that thermal equilibrium between the HDE and the horizon gets established when

$$S_{ent} \sim S_{BH} \; , \tag{13}$$

one can determine S_{ent} using (12), and than find N from (13). Noting that S_{ent} also scales

with N, one obtains[2]

$$N \sim L^2 M_{Pl}^2 \, , \tag{14}$$

a really huge number ($\sim 10^{122}$) if L is taken of order of the horizon distance at present. Within the same framework that much large N would cause problems with overproduction of gravitinos in a low-entropy post reheating epoch [18]. Also, a bound $N_{max} \simeq 10^{32}$ was obtained in alike theories by noting that we have not seen any strong gravity in the particle collisions [19] [3]. Even better limits can be inferred in the present framework when considering some particle-physics phenomena [20]. In addition, with N as a running number as given by (14), a question regarding internal consistency of the large-N framework does also arise. Namely, in the Li's model $d_E = L \sim a^{1-1/c}$ when dark energy dominates, and the horizon area is non-decreasing with time for the non-phantom stuff ($c \geq 1$). This means that in order to maintain thermal equilibrium N from (14) should grow without limit as time goes by if $c > 1$, jeopardizing thereby the internal consistency of the framework. In order to keep the internal consistency one thus has to resort to the de Sitter limit in the infinitely far future, i.e., $c = 1$. In this case N saturates asymptotically to a finite number. It is interesting to see how the internal consistency of the large-N HDE framework singles out $c = 1$ when applied to the Li's model.

If one still insists that the large-N HDE framework (or whatever other unknown mechanism) is capable to bring a fluid with $\rho_\Lambda \sim L^{-2} M_{Pl}^2$ in thermal equilibrium with the horizon near the present epoch, then one can speak for the first time of thermal (or coarse grained)

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[2] Indeed, by plugging (14) back into (12), a wanted result $\Lambda \sim L^{-1}$ is obtained.

[3] The QFT of KK particles (or equivalently for four-dimensional models with large-N species as covered in [19]) only takes cares of the UV cutoff - how it gets reduced in the presence of a large number of particle species. It provides the benchmark value ($N \simeq 10^{32}$), obtained by noting that we have not seen any strong gravity in the particle collisions. On the other hand the present QFT setup deals both with the UV and IR cutoffs, furthermore the scenario does exhibit a peculiar sort of UV/IR mixing, meaning that this way an information from quantum gravity becomes encoded in such a QFT framework (the main motivation for such a modification of the effective QFT framework being of course the compliance of the QFT with the holographic bounds). The two large-N approaches coincide only if the cutoffs coincide, i.e., if $\Lambda \simeq L^{-1}$, which is nothing but the black-hole limit (this is why the authors of [19] indicated a non-perturbative nature of their bound). It can be shown that the fine-grained entropy as given by (1) becomes N-dependent and begins to sustain the black hole entropy for the maximal N, which in the present scenario amounts $\Lambda \simeq L^{-1}$.

entropies. They can be determined with the aid of the first law of thermodynamics

$$T_{hor} dS_\Lambda = d(\rho_\Lambda V) + p_\Lambda dV \ , \tag{15}$$

where $T_{hor} = 1/(2\pi L)$ is the horizon temperature, $V = (4\pi/3)L^3$ and $p_\Lambda = w_\Lambda \rho_\Lambda$. One obtains

$$dS_\Lambda = \pi M_{Pl}^2 c^2 (1 + 3w_\Lambda) L dL \ . \tag{16}$$

Noting that $w_\Lambda = -1/3 - 2\sqrt{\Omega_\Lambda}/3c$, it can be seen that with the integration of (16) one necessarily deals with negative entropies for the non-phantom stuff ($c > 1$). In this case the horizon area grows without limit towards future and the constant of integration cannot be chosen as to make up for this negative contribution. Thus a non-phantom fluid effectively behaves as a phantom-fluid whose entropy is always negative [21]. We see again that a true CC limit in the infinitely far future ($c = 1$) is singled out. In this limit the horizon area approaches asymptotically a constant value, so the constant of integration can be appropriately chosen as to make the total contribution positive.

Finally, a note on maintaining the thermal equilibrium. The heat capacity is defined as $C_X = T(\partial S_X / \partial T)$, with X either the horizon or the HDE stuff. Since $C_{hor} = -2\pi M_{Pl}^2 L^2$ and $C_\Lambda = 2\pi c \sqrt{\Omega_\Lambda} M_{Pl}^2 L^2$, one sees that the requirement that the sum be positive boils down to $c\sqrt{\Omega_\Lambda} > 1$. This shows that only the case away from the true CC limit ($c > 1$) is relevant for maintaining thermal equilibrium.

In conclusion, we have found out that the effective QFT framework, underlying all HDE models, is not capable to describe the holographic dark energy component in thermal equilibrium with the cosmic event horizon around the present time, a process which can be successfully described phenomenologically. The dark energy component inside the horizon would have to have the enormously larger entropy as well as the enormously smaller temperature than what is consistent with the underlying theoretical setup. When the framework is developed with a large number of particle species, the dark energy entropy tends to increase with N while the UV cutoff (a measure of the temperature) tends to decrease with N. The present-day cosmological requirement on N is however so huge to be consistent with other phenomenological constraints on the number of particle species. In addition, the UV cutoff is so hugely diminished that such a QFT is not capable to describe virtually any relevant physics. For instance, such a theory is not capable to describe even thermal photons of the universe at present, at a temperature $\sim 10^{-4}$ eV. If one is contented with the

phenomenological description of the process only, then this would entail a problem where negative entropies do arise for the non-phantom component. The origin of this problem lies in the fact that the energy density of the HDE is devoid of a true constant term [22]. Unfortunately, holography always does away with such a constant term in the energy density. In contrast, renormalization-group running cosmologies [23], besides having the same variable part of the energy density as the HDE component, are always accompanied with such a term. The constant serves to prevent the horizon area to grow with time without limit. For the present case this is only the case for a singular point ($c = 1$) in the parameter space. For another argument supporting c=1 in the far future, see [24]. On the other hand, maintaining thermal equilibrium with the horizon is not consistent with this point. The internal inconsistencies in HDE models found here adds to the previous ones having been discussed earlier [25].

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